On Simultaneous Transport of Heat and Mass in Natural Convection

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The authors have studied turbulent thermohaline natural convection from several points of view (*1, 2, 3*). The emphasis of this work has been on the overstable case of steady state natural convection between porous horizontal flat plates in which both heat and salt are added from below and removed at the upper plate. This combination is not the only one possible in thermohaline convection, but it is an advantageous choice for first study because of its similarity to the widely studied Rayleigh-Jeffreys problem of thermal natural convection. In this combined problem, addition of salt from the bottom tends to stabilize the system gravitationally and thus inhibits heat transfer as compared with thermal natural convection. One would then find occasion to compute the rates of heat and salt transfer as functions of applied differences of temperature and concentration. The present authors have made a prediction of these rates; after introduction of some terms the results will be reviewed and extensions made to other natural-convection problems involving transport of both heat and mass. Description of these extensions is the purpose of this note.

The usual Rayleigh number will be referred to as the thermal Rayleigh number, and there is an analogous concentration Rayleigh number; the definitions are

$$N_{Ra_T} = \frac{g\beta_T \, \Delta T L^3}{\nu D_T}, \quad N_{Ra_c} = \frac{g\beta_c \, \Delta C L^3}{\nu D_c} \qquad (1)$$

The fluxes arising from these driving parameters may be written in terms of the Nusselt and Sherwood numbers

$$N_{Nu} = \frac{h_T L}{k_T} = \frac{F_H}{k_T \dfrac{\Delta T}{L}} \quad \text{and} \quad N_{Sh} = \frac{h_c L}{D_c} = \frac{F_c}{D_c \dfrac{\Delta C}{L}} \qquad (2)$$

The usual property groups N_{Pr} and N_{Sc} are likely to appear in problems of this kind, and it has in fact been found advantageous to use their ratio, the Lewis number $N_{Le} = D_T/D_c$. While several other combinations of the driving parameters and property groups may be used (*1, 3*), one combination which illustrates the effects of interaction between ΔT and ΔC clearly is N_{Ra_T}, N_{Ra_c}, N_{Pr}, N_{Le} for the independent groups and N_{Nu}, N_{Sh} for the dependent variables.

The prediction of N_{Nu} and N_{Sh} for the overstable case involves a development of the variational method which has lately become fashionable in turbulent fluid mechanics (*1, 2*). Briefly, one forms integral moments of the governing Boussinesq equations with the fluctuating components of velocity, temperature, and concentration. The assumptions of a statistical steady state and horizontal homogeneity are introduced to simplify the resulting integral equations. Application of a variational technique leads to upper bounds on the Nusselt and Sherwood numbers, subject to suitable physical constraints. The results are

$$(N_{Nu} - 1) = .14[N_{Ra_T} - N_{Ra_c} \, N_{Le}^{-16/11}]^{3/8} \qquad (3)$$

and

$$\frac{N_{Sh} - 1}{N_{Nu} - 1} = N_{Le}^{6/11} \qquad (4)$$

For the special case of thermal natural convection, Equation (3) provides a reasonable (*4, 5*) prediction in the turbulent ($N_{Ra_T} > 50{,}000$) regime. Although the prediction is high, the functional form is correct, and the Lewis number and the stabilizing effect of the solute gradient both appear in a physically defensible way. The range of N_{Ra_c} which is suitable has never been determined, but it has been suggested (*1*) that the maximum N_{Ra_c} for which this theory is likely to be valid is $N_{Le}^{16/11}$ times the thermal Rayleigh number, N_{Le} being about 100 for salt-water solutions. Experiments in this area are being undertaken by the authors.

Algebraic signs in Equation (3) depend on the directions of the thermal and concentration gradients or, alternatively and more appropriately for turbulent situations, on the *drops* in temperature and concentration between the horizontal plates. From the viewpoint of gravitational stability, the temperature drop is destabilizing and the concentration drop stabilizing for the overstable thermohaline convection already described. There are two other possibilities for convection, stabilizing temperature drop with destabilizing concentration drop, and destabilizing temperature drop with destabilizing concentration drop. The fourth possible combination, both drops stabilizing, will not allow convection and has its solution in molecular effects only. These combinations of stability influences arise from two types of inputs to the stability effect of a drop of temperature or concentration. The first input is the algebraic sign of the drop (or decrease from bottom to top) of T or C. This is the imposed combination of boundary conditions, independent of the materials used. The second input is the algebraic sign of the rate of density change with temperature or concentration $(\partial\rho/\partial T)_{p,C}$ or $(\partial\rho/\partial C)_{p,T}$. Each of these partials is a property of the solution or its components. It is common for $\partial\rho/\partial T$ to be negative, but in certain instances, such as for water below 4C, it is positive. Likewise it is common for $\partial\rho/\partial C$ to be positive for dense solutes in light solvents, but there are many examples for which this derivative is negative.

There are a total of twelve possible combinations of algebraic signs on ΔT, ΔC, $\partial\rho/\partial T$, $\partial\rho/\partial C$ leading to the three possible stability combinations. These possibilities are listed in Table 1 along with an example of each combination. The examples have been selected mainly to illustrate the concept, but direct applications of several of them to chemical engineering processes are evident. Only the distinctions among the three stability combinations, Cases I, II, and III, are needed to show the proper modifications

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TABLE 1. THE TWELVE POSSIBILITIES FOR THERMOSOLUTAL NATURAL CONVECTION

Case	Influence of temperature drop on density rise	Influence of concentration drop on density rise	Sign on ΔT	Sign on ΔC	Sign on $\partial\rho/\partial T$	Sign on $\partial\rho/\partial C$	Examples
I	(+) Destabilizing	(−) Stabilizing	+	+	−	+	Overstable thermohaline natural convection, evaporation or sublimation of a heavy gas from a warm surface
	(+) Destabilizing	(−) Stabilizing	+	−	−	−	Absorption of a light solute onto a warm surface
	(+) Destabilizing	(−) Stabilizing	−	+	+	+	Freezing at the bottom of a saltwater system, leaving salt at the bottom surface
	(+) Destabilizing	(−) Stabilizing	−	−	+	−	Freezing at the bottom of a tank with absorption of a light solute
II	(+) Destabilizing	(+) Destabilizing	+	+	−	−	Evaporation or sublimation of a light gas from a warm surface
	(+) Destabilizing	(+) Destabilizing	+	−	−	+	Evaporative cooling at free surface of a pool of liquid, leaving heavy solute behind
	(+) Destabilizing	(+) Destabilizing	−	+	+	−	Freezing at the bottom of a tank with release of a light solute
	(+) Destabilizing	(+) Destabilizing	−	−	+	+	Freezing at the bottom of a tank with absorption of a heavy solute
III	(−) Stabilizing	(+) Destabilizing	+	+	+	−	Melting at the bottom of a tank with release of a light solute from the wall
	(−) Stabilizing	(+) Destabilizing	+	−	+	+	Penetrative convection of saltwater with freezing at the top and residual solute
	(−) Stabilizing	(+) Destabilizing	−	+	−	−	Evaporation or sublimation of a light vapor into a heavy gas above a cold wall
	(−) Stabilizing	(+) Destabilizing	−	−	−	+	Free-convection regime in oceanic thermocline

TABLE 2. NUSSELT-NUMBER PREDICTIONS FOR THREE CASES OF THERMOHALINE NATURAL CONVECTION

Case	Temperature drop, ΔT	Concentration drop, ΔC	Upper-bound prediction of Nusselt number
I.	Destabilizing	Stabilizing	$N_{Nu} - 1 = .14[N_{RaT} - N_{Rac} N_{Le}^{-16/11}]^{3/8}$
II.	Destabilizing	Destabilizing	$N_{Nu} - 1 = .14[N_{RaT} + N_{Rac} N_{Le}^{-16/11}]^{3/8}$
III.	Stabilizing	Destabilizing	$N_{Nu} - 1 = .14[- N_{RaT} + N_{Rac} N_{Le}^{-16/11}]^{3/8}$

of Equation (3) which accommodate the various input conditions. In each case, the Rayleigh numbers are to be thought of as positive numbers; that is, the absolute value is to be used. Destabilizing Rayleigh numbers (of either type, concentration or thermal) are then made to carry positive algebraic signs in Equation (3), and stabilizing Rayleigh numbers are made to carry negative signs. The authors have confirmed in the rather lengthy analysis of reference 2 that this simple sign-change procedure results from the indicated stability combinations, Case I having been the subject of reference 2. The resulting predictions are shown in Table 2. For each case the Sherwood number remains that predicted by Equation (4).

For systems whose Lewis numbers are near unity, the upper-bound predictions reduce to the well known Lewis relationship, Nusselt number = Sherwood number. There are few analytical predictions or experimental results for Lewis numbers far from unity. The predictions of Table 2 are presented as forms to be compared with data for systems of large or small Lewis number.

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NOTATION

C = concentration
D_c, D_T = molecular diffusivities of mass and heat
F_c, F_H = fluxes of mass and heat
g = acceleration of gravity
h_c, h_T = Newtonian surface conductances of mass and heat
k_T = thermal conductivity
L = depth of section
T = temperature
p = pressure
β_c, β_T = coefficients of volume expansion due to changes of concentration or temperature
ΔC, ΔT = decrease in concentration and temperature from bottom to top

ν = kinematic viscosity
ρ = density
N_{Le} = Lewis number
N_{Nu} = Nusselt number
N_{Pr} = Prandtl number
N_{Ra_c} = concentration Rayleigh number, Equation (1)
N_{Ra_T} = thermal Rayleigh number, Equation (1)
N_{Sc} = Schmidt number
N_{Sh} = Sherwood number

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Fixed Bed Sorption with Recycle

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There are several types of processing operations in which a stream is taken from a source, fed to a sorbent bed, and then recycled back to the source. One such situation is shown in Figure 1, where a sorbent bed is being used to remove a desired product *B* from a reacting mixture. Unreacted component *A* is recycled back to the reactor. Besides recovering product, the sorbent bed ensures that the reaction mixture remains low in *B*. Therefore, if the reaction is reversible, the reverse reaction is suppressed and nearly complete conversion of *A* is obtained.

A second example of sorption with recycle is the sorbent artificial kidney system discussed by Cooney (*2*) and Cooney and Mace (*3*). The sorbent bed in this case consists of microcapsules containing activated carbon particles, ion exchange resin particles, and urease enzyme solution. This type of device, which has been developed in rudimentary form and is now beginning to be tested on animals (*1*) would remove the major toxic metabolites (urea, uric acid, creatinine) from the blood of a person suffering from lack of kidney function.

In recycle sorption systems, the economic point for bed replacement or regeneration often is not the point at which the first detectable breakthrough of the sorbable species is observed. Rather it may be the time when maximum capacity of the bed has been reached, that is, when more solute is leaving the bed than is entering. In any event, whether breakthrough or maximum capacity is the chosen criterion, it is necessary to determine the shape of the solute concentration profile in the bed as a function of time in order to establish when the sorber should be taken off stream. Unfortunately, no general analytical theories exist which give the proper profile descriptions when there is recycle (that is, when the boundary condition at the bed inlet is a function of time). Analytical solutions *may* be possible for linear equilibria, but have not yet been obtained.

The most general approach to solving such problems is via numerical integration of the pertinent differential equations. For the sorbent bed we have, neglecting axial dispersion

$$v\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \frac{1-\epsilon}{\epsilon}\frac{\partial q}{\partial t} = 0 \tag{1}$$

$$\frac{\partial q}{\partial t} = f(q^*, q, c^*, c) \tag{2}$$

$$q^* = f(c) \tag{3}$$

These are the continuity equation for the solute, a mass transfer rate law, and an equilibrium distribution function, respectively. For the source we would have, if it were a single perfectly mixed tank

$$F(c_{\text{out}} - c_{\text{in}}) + R = V\frac{dc_{\text{in}}}{dt} \tag{4}$$

where *R* is a production term (sometimes zero). Note that the subscripts here refer, as shown in Figure 1, to the *bed* inlet and outlet concentrations, not to the *source* inlet and outlet concentrations. Because the behavior of sorption systems with recycle has not been considered in any detail in the previous literature, and because there are features about such systems which are unique and interesting, we wish to present information for the case of a reaction product recovery system and thereby illustrate the basic phenomena involved.

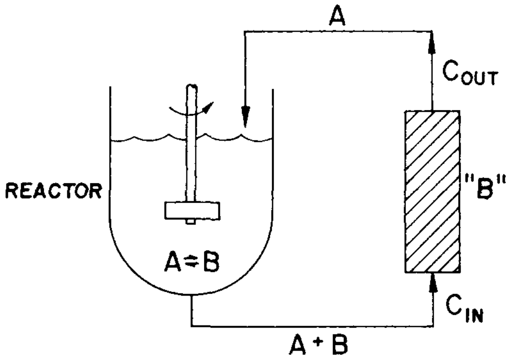

Fig. 1. Reactor-sorber system with recycle.